UDC 004.94

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A POSSIBLE UTILITY-BASED EXPLANATION OF DEATON'S PARADOX (AND HABITS OF MIND)

What is Deaton's paradox. Angus Deaton, the winner of the 2015 Nobel Prize in Economics, became well-known after his seminal 1989 paper [1]. This paper of Deaton empirically analyzed how consumption depends on income level.

Until this study, it was believed that when the income increases, the consumption increases as well. The paper [1] showed that, contrary to this belief, an increase in income does not immediately lead to an increase in consumption. The increase in consumption does follow, but after some delay. As a result, even when the income changes abruptly, the resulting change in consumption is much smoother.

This counterintuitive behavior is known as *Deaton's paradox*.

So far, there is no well-accepted explanation of Deaton's paradox. Several researchers – starting with Deaton himself – attempted to explain this paradox. However, until now, none of the proposed explanations was convincing enough to be well accepted.

What we do. In this abstract, we propose a possible explanation of Deaton's paradox based on the most fundamental notion of decision theory: the notion of utility.

Consumption: a simplified description. Let us start with a simplified model that describes how we make a decision on what to consume. Whether we talk about food or clothes or car or an apartment to rent, there are usually several options ranging both in cost and in value. Once we have allocated a certain amount of money c_0 to this task, we select, among all options i with cost c_i not exceeding c_0 , the option with the highest value v_i to us.

In precise terms, we have a list of options. Each option i (i = 1, 2, ..., n) is characterized by a pair of numbers (c_i, v_i) , where c_i is

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the option's cost and v_i is the option's value. We select the option i that maximizes v_i under the constraint $c_i \leq c_0$, i.e., the option with the value v_0 corresponding to c_0 .

Towards a more realistic description of consumption. The above description does not take into account that our utility depends not only the current value v(t), but also on our memories of past consumptions. These memories fade with time, so the value experienced s moments ago is added with a weight w(s) depending on s. The resulting utility u(t) is thus equal to $u(t) = v(t) + \sum_{s=1}^{\infty} w(s) \cdot v(t-s)$.

Memory passes through intermediate moments of time: what was remembered after s time periods with a weight w(s) after period s' get further decreased by a multiplicative weight w(s'), so

$$w(s+s') = w(s) \cdot w(s').$$

Since $s = 1 + \ldots + 1$ (s times), this implies that $w(s) = q \cdot \ldots \cdot q$ (s times), where $q \stackrel{\text{def}}{=} w(1)$, thus $w(s) = q^s$ for some q < 1.

An even more realistic description appears if we take into account that experiencing the same thing (same food, etc.) enhances the memory. This enhanced memory can be described by using a different "forgetting" coefficient Q > q. The final formula is thus

$$u(t) = v(t) + \sum_{s: \ v(t-s) = v(t)} Q^s \cdot v(t-s) + \sum_{s: \ v(t-s) \neq v(s)} q^s \cdot v(t-s).$$

How this more realistic description affects the actual consumption. We choose an alternative for which the utility is the largest. Suppose that for a long time, we had a fixed income level c_0 , and we used an alternative with value v_0 . Then, if the income suddenly increases to $C_0 > c_0$, we can either stay with the previous option or use a new option and switch to a new alternative with a larger value $V_0 > v_0$ whose cost $C_0 > c_0$ we can now afford. In the second case, the utility is equal to

$$u'' = V_0 + \sum_{s=1}^{\infty} q^s \cdot v_0 = V_0 + \frac{q}{1-q} \cdot v_0.$$

In the first case, the utility is equal to

$$u' = v_0 + \sum_{s=1}^{\infty} Q^s \cdot v_0 = \frac{1}{1 - Q} \cdot v_0.$$

In the extreme case when $Q \approx 1$ and $q \approx 0$, we have $u'' \approx V_0$ and $u' \approx \infty$, so clearly $u' \gg u''$ and it makes sense for the consumer to stay with the original consumption choice – and thus, not to increase his/her consumption. Same is true for slightly smaller Q and slightly larger q.

Comment. Such an inertia is well known in education, it is called habits of mind; see, e.g., [2].

So why does consumption eventually increase? At first glance, it may seem that the same situation will reappear again and again, and the consumer will never switch to a better option. A detailed analysis shows, however, that eventually, there will be a switch.

Indeed, because of the inertial choice, out of the amount C_0 allocated for this particular type of consumption, only about c_0 is used. So, the difference $C_0 - c_0$ can be re-invested, and next time period, instead of the amount C_0 , we have a larger amount $C = C_0 + k \cdot (C_0 - c_0)$, where k is the interest on this investment. After T periods like this, we will have the amount $C_0 + k \cdot T \cdot (C_0 - c_0)$. As T increases, this amount tends to infinity, so the consumer can afford more and more valuable options with an increasing value V. Eventually, we will have $V + \frac{q}{1-q} \cdot c_0 > \frac{1}{1-Q} \cdot c_0$. In this case, the customer will eventually switch to spending more – with a delay, which is now properly explained.

Acknowledgments. This work was supported by the Center of Excellence in Econometrics, Faculty of Economics, Chiang Mai University, Thailand, and by the National Science Foundation grants HRD-0734825, HRD-1242122, and DUE-0926721.

The authors are thankful to the participants of the 4th International Symposium on Integrated Uncertainty in Knowledge Modeling and Decision Making IUKM'2015 (Nha Trang, Vietnam, October 15–17, 2015) for valuable discussions.

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