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HOW TO MAKE SURE THAT EVERYONE WORKS TOWARDS A COMMON GOAL: TOWARDS OPTIMAL INCENTIVES

Formulation of the problem. A company as a whole is interested in increasing its profit. It is therefore desirable to com up with incentives that encourage everyone to work toward the common goal.

This problem is not easy. In spite of all the efforts to come up with reasonable incentives, the existing incentives schemes are not perfect. Everyone who has worked for a large company knows of turf wars and other problems, as a result of which divisions within the company often hurt each other instead of productively working towards a common goal.

Stock incentive: a seemingly reasonable solution. At first glance, a natural way to make everyone interested in the common goal is to make the incentive proportional to the company's success. This can be achieved, e.g., by giving employees stock options as part of their salaries: this way, the better off the company, the largest the actual salary.

Alas, this seemingly reasonable idea often does not work. At first glance, the stock incentive idea should work well. But imagine a typical employee of a very large company. His/her success contributes to only a tiny portion of the company's profit. So, whether this employee does not do anything or works really hard, the overall profit practically does not change – and thus, the employee's salary does not change.

So, the stock option scheme is actually a *disincentive*: if we take into account the efforts needed to work hard, the employees are thus encouraged to do nothing – and therefore, the company's profits decrease. So, what is a good incentives scheme?

Our proposal: main idea. Our proposal is that instead of making the salary proportional to the overall company profit, we should make it proportional to the person's contribution to this profit.

Our proposal: discussion. It is important to take into account the effect of the person's contribution under the assumption that everyone else works optimally. For example, a worker who produces a certain number of gadgets should be rewarded because if the sales department works well, these gadgets will bring extra credit to the company. However, if the sales department is not functioning well and the gadgets are not sold, the company only experiences a loss – loss of materials, expenses needed for storing the gadgets, etc.

This idea deals with everyone who is necessary for the company's success: workers, janitors, accountants, investors, managers, etc.

Let us formulate this idea in precise terms.

Our proposal: a description in precise terms. Let us first introduce some notations:

- let n be the total number of employees,
- let X_i be the set of all possible actions of the i-th employee,
- let $0 \in X_i$ be the case when the i-th employee does not do anything,
- let $p(x_1, \ldots, x_n)$ be the profit gained by the company when each employee i performs action x_i, and
- let $x_i^{\rm act} \in X_i$ be the actual action performed by the i-th employee.

In these notation, the reward $r_i(x_i^{\rm act})$ to the i-th employee should be equal to

$$r_i(x_i^{\rm act}) =$$

$$\max_{x_1 \in X_1, \ldots, x_{i-1} \in X_{i-1}, x_{i+1} \in X_{i+1}, \ldots} p(x_1, \ldots, x_{i-1}, x_i^{\rm act}, x_{i+1}, \ldots, x_n) -$$

$$\max_{x_1 \in X_1, \ldots, x_{i-1} \in X_{i-1}, x_{i+1} \in X_{i+1}, \ldots} p(x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_n). \quad (1)$$

Comment. This idea is similar to the main idea behind the Shapley vector, which is determined in terms of the differences $v(S \cup \{i\}) - v(S)$ between what the coalition S can get for itself and what it can get if it collaborates with the i-th participant; see, e.g., [1].

This proposal indeed leads to the optimal global solution. Indeed, let us assume that there is exactly one combination of strategies $(x_1^{\rm opt}, \ldots, x_n^{\rm opt})$ that results in the maximal profit, i.e., for which

$$p(x_1^{\rm opt}, \ldots, x_n^{\rm opt}) =$$

$$\max_{x_1 \in X_1, \ldots, x_{i-1} \in X_{i-1}, x_i \in X_i, x_{i+1} \in X_{i+1}, \ldots} p(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n).$$

In this case, the following statement holds.

Proposition 1. *In situations when there is only combination of actions that leads to the optimal solution, if every employee maximizes the value $r_i(x_i)$, the the resulting combination of strategies is optimal.*

Proof. Since the second term in $r_i(x_i)$ does not depend on x_i, the largest possible value of $r_i(x_i)$ is attained when the first term is the largest, i.e., when the profit $p(x_1, \ldots, x_n)$ attains its largest possible value. By our assumption, this is exactly when $x_i = x_i^{\rm opt}$. The statement is proven.

Discussion: what do we do if there are many optimizing combinations of strategies? It may happen that there are several combinations of strategies that lead to optimal solution. For example, suppose that we are running a bus company in a small town where there are two bus drivers and two bus routes. The optimal solution is when:

- either driver A runs route B and driver B runs route A,
- or driver A runs route A and driver B runs route B.

However, if both driver A and B run the same route A, then for both the reward will be optimal but the overall profit will be a disaster, since no one runs route B.

To avoid such situations, we need an additional coordination between workers, which will rearrange the sets X_i to guarantee the desired uniqueness.

But can we afford this solution? Absolutely: if everyone works fine, every worker get exactly what he or she contributed.

Comment. Of course, if someone screwed up and the company loses money, we may not have any profit to divide between the employees: but this is true irrespective of how we decide to divide the profit.

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